HYDRODYNAMICS AND HEAT TRANSFER IN A LAYER OF LIQUID ON A ROTATING SURFACE, ALLOWING FOR INTERACTION WITH A GAS FLOW

N. S. Mochalova, L. P. Kholpanov, and V. Ya. Shkadov

UDC 532.526.75

The hydrodynamics and heat transfer of a layer of liquid on a rotating surface are analyzed theoretically on the boundary-layer approximation under conditions in which a gas flow interacts with the liquid film.

The hydrodynamics and mass transfer of a layer of liquid on a rotating surface were studied in the absence of wave formation and frictional forces at the interface in our earlier paper [1]. In this paper the same problem will be solved with due allowance for frictional forces at the interface, and the heat transfer from the liquid film to the rotating surface will be calculated under these conditions.

1. Let the x axis signify the arc length along the flooded wall of a spiral channel and y, the distance from the wall along the normal. We assume that the liquid is incompressible, the motion steady, and the flows isothermal. The thin layer of liquid moves without wave formation along an Archimedes spiral, which in polar coordinates r, θ obeys the equation $r = A\theta$, $A > 0$. We also assume that the pressure gradient in the layer arises solely from the rotation. Under these assumptions the motion of the thin layer of liquid may be described by the same Prandtl equations as in [1]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=X-\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}+\gamma\frac{\partial^2 u}{\partial y^2},$$
$$-\frac{u^2}{R(x)}=Y-\frac{1}{\rho}\cdot\frac{\partial p}{\partial y},\tag{1}$$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0;$$

R(x) is the radius of curvature in polar coordinates:

$$R(x)=\frac{(r^2+r_\theta^2)^{3/2}}{r^2+2r_\theta^2-rr_{\theta\theta}}=\frac{A(\theta^2+1)^{3/2}}{\theta^2+2},$$

where

$$r_\theta=\frac{dr}{d\theta},\quad r_{\theta\theta}=\frac{d^2r}{d\theta^2};$$

and X, Y are the projections of the mass forces on the x and y axes respectively. The mass forces acting on the particles of liquid include the centrifugal force $\bar{F}_c = \omega^2 R(x)$ and the Coriolis force of inertia $\bar{F}_{cor} = 2\bar{\omega}\times\bar{V}$. A change in the direction of rotation of the spiral is only reflected in the second of these. The projections of the mass forces on the x and y axes take the form

$$X=\omega^2R(x)\cos\alpha\pm2\omega v,$$
$$Y=\omega^2R(x)\sin\alpha\pm2\omega u,\tag{2}$$

where the upper sign corresponds to the anticlockwise, and the lower sign to the clockwise, rotation of the spiral; α is the angle made by the centrifugal force vector with the positive direction of the tangent, since

Institute of New Chemical Problems, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 4, pp. 684-690, October, 1976. Original article submitted August 5, 1975.

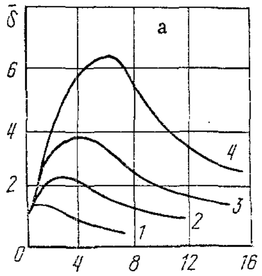

Fig. 1

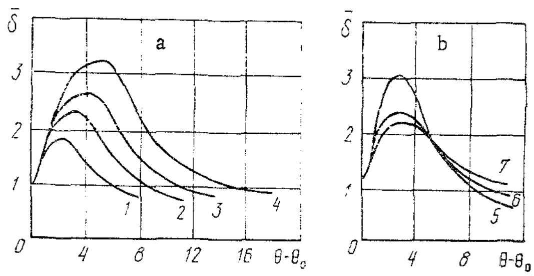

Fig. 2

Fig. 1. Dimensionless thickness of the liquid film as a function of the length of the spiral for Re = 300; Γ1 = 1: a) for E5 = 1, 1) E1 = 0.4; 2) 1; 3) 1.6; 4) 2.5; b) for E1 = 1, 5) E5 = 0.5; 6) 1; 7) 1.5.

Fig. 2. Dimensionless thickness of the liquid film as a function of the length of the spiral for E1 = 1; E5 = 1; a) for Γ1 = 1, 1) Re = 100; 2) 300; 3) 500; 4) 1000; b) for Re = 300, 5) Γ1 = 2; 6) 1; 7) 0.

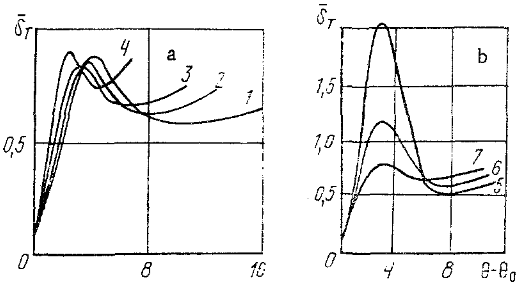

Fig. 3. Dimensionless thickness of the thermal boundary layer as a function of the length of the spiral for E1 = 1; E5 = 1; Pr = 10: a) for Γ1 = 0, 1) Re = 1000; 2) 500; 3) 300; 4) 100; b) for Re = 300, 5) Γ1 = 0; 6) 1; 7) 2.

$$\operatorname{tg}\alpha = \frac{r}{r_\theta} = \theta, \text{ then } \cos\alpha = \frac{1}{\sqrt{\theta^2+1}}, \quad \sin\alpha = \frac{\theta}{\sqrt{\theta^2+1}}.$$

In the variables θ, y the system of Eqs. (1) may be expressed as follows:

$$\frac{u}{A\sqrt{\theta^2+3}} \cdot \frac{\partial u}{\partial \theta} + v\frac{\partial u}{\partial y} = X - \frac{1}{\rho} \cdot \frac{1}{A\sqrt{\theta^2+1}} \cdot \frac{\partial p}{\partial \theta} + \gamma\frac{\partial^2 u}{\partial y^2},$$

$$-\frac{u^2}{R(\theta)} = y - \frac{1}{\rho} \cdot \frac{\partial p}{\partial y}, \tag{3}$$

$$\frac{1}{A\sqrt{\theta^2+1}} \cdot \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial y} = 0.$$

It follows from the condition of attachment to the wall that

$$\text{for} \quad y = 0, \quad u = v = 0. \tag{4}$$

The effect of the gas flow on the flow of the thin liquid layer is taken into account by way of the tangential forces on the interface, i.e.,

$$\text{for} \quad y = \delta, \frac{du}{dy} = \frac{\tau_0}{\mu} = B, \; p = p_a = \text{const}, \; u = U. \tag{5}$$

We solve system (3) by the method of integral relationships. The polynomial of the second degree which satisfies boundary conditions (4) and (5) takes the form

$$\frac{u}{U} = \left(2 - \frac{B\delta}{U}\right)\frac{y}{\delta} - \left(1 - \frac{B\delta}{U}\right)\left(\frac{y}{\delta}\right)^2, \tag{6}$$

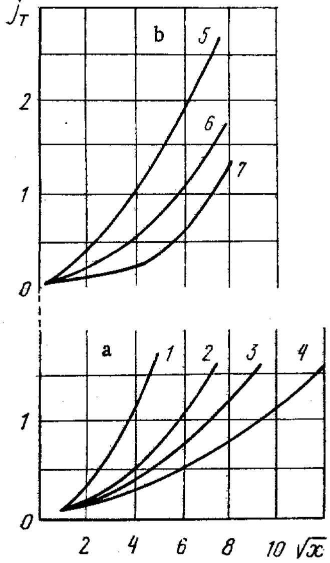

Fig. 4. Relationship between j_T and the length of the spiral for Re = 300; Pr = 10: a) for Γ1 = 0, E5 = 1, 1) E1 = 0.4; 2) 1; 3) 1.6; 4) 2.5; b) for Γ1 = 1, E1–1, 5) E5 = 0.5; 6) 1; 7) 1.5.

where U is determined from the equation for the rate of flow $\int_0^{\delta(x)} u dy = q = \text{const}$ and is expressed by

$$U=\frac{3}{2}\cdot\frac{q}{\delta}+\frac{B\delta}{4}. \tag{7}$$

Eliminating the pressure from the second equation of system (3) we find $\partial p/\partial\theta$. After integrating the first equation of system (3) with respect to y over the boundary layer and taking account of the $\partial p/\partial\theta$ just found, we then obtain a nonlinear equation of the first order in $\delta(\theta)$. In dimensionless coordinates this takes the form

$$\frac{d\delta}{d\theta}=\left\{\frac{9\mathrm{Ga}}{\mathrm{Re}^2E5}\cdot\frac{(\theta^2+1)^{3/2}}{\theta^2+2}\bar{\delta}-\frac{9}{2}\bar{\delta}^2\frac{\mathrm{Ga}}{\mathrm{Re}^2}\cdot\frac{\theta^4+5\theta^2+2}{(\theta^2+2)^2}\right.$$
$$\left.-\frac{E5\theta(\theta^2+4)}{(\theta^2+1)^{5/2}}\left(\frac{33}{40}+\frac{3}{40}\bar{B}\bar{\delta}^2+\frac{1}{160}\bar{B}^2\bar{\delta}^4\right)+\frac{9\sqrt{\theta^2+1}}{\mathrm{Re}E5}\left(\frac{\bar{B}}{2}+\frac{1}{\bar{\delta}^2}\right)\right\}$$
$$\times\left\{-\frac{6}{5}\cdot\frac{1}{\bar{\delta}^2}+\frac{\bar{B}}{20}+\frac{\bar{B}^2\bar{\delta}^2}{40}\mp\frac{6\mathrm{Ga}^{1/2}E5^{1/2}}{\mathrm{Re}}\left(\frac{7}{8}+\frac{3}{16}\bar{B}\bar{\delta}^2\right)+\frac{9\mathrm{Ga}}{\mathrm{Re}^2}\right.$$
$$\left.\times\frac{\theta(\theta^2+1)}{\theta^2+2}\cdot\bar{\delta}-\frac{E5(\theta^2+2)}{(\theta^2+1)^{5/2}}\left(\frac{3}{20}\bar{B}\bar{\delta}+\frac{1}{40}\bar{B}\bar{\delta}^2\right)\right\}^{-1}, \tag{8}$$

where the upper sign corresponds to anticlockwise and, the lower to clockwise, rotation. We note that for B = 0, i.e., in the absence of interaction between the gas flow and the layer of liquid, Eq. (8) takes the form derived in our earlier paper [1]. Equation (8) may be solved numerically by the Runge–Kutta method.

The characteristic form of the relationship between the thickness of the liquid film on the spiral and the length of the latter for various values of the hydrodynamic parameters is shown in Fig. 1a, b and Fig. 2a, b.

We see from these figures that for all operating conditions of the spiral apparatus considered the thickness of the liquid film passes through a maximum as the parameters are varied and then falls, with a tendency to approach a constant value. The rate at which the film thickness approaches a constant value depends on Re, Ga, B, E5. The greater Ga and the smaller Re and E5, the more rapidly does the equation of the apparatus approach the situation involving a constant film thickness.

The distance $\bar{x}_k$ such that, after traversing it in a downstream direction, the film thickness and surface velocity differ by 0.1% from a constant value (i.e., the length of the "inlet" section), may be found from the numerical solution of Eq. (8) and takes the form

$$\bar{x}_k = (0.16\,\mathrm{Re} + 15)\sqrt[3]{\frac{\mathrm{Re}}{\mathrm{Ga}}}\,. \tag{9}$$

The frictional force τ of the liquid film on the surface of the whole length of the spiral channel has the form

$$\tau = \int_0^L \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} dx = 1{,}44\rho\gamma^{1/3}q^{1/3}\omega^{4/3}A^{2/3}L\left[\exp\left(-\frac{0.26\tau_0}{\rho\nu^{1/3}q^{1/3}\omega^{4/3}A^{2/3}}\right)\right]^{-2} - \frac{\tau_0 L}{2}\,. \tag{10}$$

2. If the heat-transfer resistance is concentrated in the liquid phase, the heat-transfer coefficient from the liquid film to the wall of the spiral heat-exchanger may be calculated from the energy equation for the liquid layer:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2}\,, \tag{11}$$

in which the velocities u and v are regarded as determined from the solution of the hydrodynamic problem (6).

We assume that the flow of the liquid film is laminar, the thermal diffusivity is constant, and the dissipation energy is small enough to be neglected. In solving problem (11) the boundary conditions take the form

$$\text{for } y = 0 \quad T = T_\mathrm{w}\,, \quad \text{for } y = \delta_T \quad T = T_\mathrm{f}\,. \tag{12}$$

We introduce the dimensionless temperature $\bar{T} = (T - T_W) \times (T_f - T_W)^{-1}$. A polynomial of the third degree satisfying the boundary conditions (12) is as follows:

$$\bar{T} = \frac{3}{2}\left(\frac{y}{\delta_T}\right) - \frac{1}{2}\left(\frac{y}{\delta_T}\right)^3.$$

Integrating with respect to y from y = 0 to y = δ_T, i.e., over the thickness of the thermal layer, and allowing for the boundary conditions (12), we obtain a nonlinear equation of the first order in $\delta_T(\theta)$. In dimensionless coordinates this takes the form

$$\frac{d\bar{\delta}_T}{d\theta} = \left[-\frac{720\sqrt{\theta^2+1}}{\mathrm{Re}\,E5\,\mathrm{PR}} - \frac{d\bar{\delta}}{d\theta}\left(\frac{96\bar{\delta}_T^3}{\bar{\delta}^3} - \frac{30\bar{\delta}_T^4}{\bar{\delta}^4} + \frac{5B\bar{\delta}_T^4}{\bar{\delta}^2}\right)\right]\left[\frac{30\bar{\delta}_T^3}{\bar{\delta}^3} - \frac{96\bar{\delta}_T^2}{\bar{\delta}^2} + 16B\bar{\delta}_T^2 - \frac{15B\bar{\delta}_T^3}{\bar{\delta}}\right]^{-1}, \tag{13}$$

in which $\bar{\delta}$ and $d\bar{\delta}/d\theta$ are to be taken from the solution to the hydrodynamic problem (8). The solution to Eq. (13) is conveniently obtained for two cases: a) in the inlet section; b) as $d\delta/d\theta \to 0$, neglecting the terms in Eq. (13) containing the dimensions of the thermal layer to the third power, i.e., in the stabilization region.

Equation (13) may be solved numerically by the Runge–Kutta method at the same time as Eq. (8). The characteristic form of the relationship between the thickness of the thermal layer on the spiral and the length of the spiral for various values of the hydrodynamic parameters is shown in Fig. 3a and b.

We see from Fig. 3 that, for the operating conditions of the apparatus under consideration, the thickness $\bar{\delta}_T$ first rises to a maximum by virtue of the fact that the thickness of the liquid film has a transient form in the inlet section; it then falls, and subsequently starts rising again.

The resultant numerical values of the thickness of the thermal layer are used to determine the average coefficient of heat transfer to the film of liquid in the inlet section. To this end we consider a certain characteristic length L and average the thermal flux on the surface of the spiral

$$\beta_T = \frac{1}{x_k}\int_0^{x_k} a\left(\frac{\partial T}{\partial y}\right)_{y=0} dx = \frac{3a}{2x_k}\int_0^{x_k}\frac{dx}{\delta_T}\,. \tag{14}$$

In Eq. (14) the averaging is carried out over the region of development of the liquid film defined by Eq. (9). An approximating formula for the heat-transfer coefficient found by numerical integration takes the form

$$\beta_T = 1.5\,\mathrm{Re}^{1/2}E5^{1/2}\nu^{1/2}a^{1/2}\bar{x}_k^{-1}j_T\,, \tag{15}$$

where

$$j_T = \exp\{2.3\sqrt{\bar{x}_k}\,\exp(-0.511E1 - 1.236) + [(-0.045\Gamma 1 - 0.45)\ln\mathrm{Re} + 0.92]\,E5 - 0.22\ln\mathrm{Pr} + 0.414E1 - 0.69\}.$$

The accuracy of the resultant approximating equations assessed by comparison with the numerical solution in the range of parameters studied amounts to about 10%.

The characteristic form of the relationship between j_T in the liquid film and the length of the spiral for various values of the hydrodynamic parameters is shown in Fig. 4a, b.

We see from these figures that, for the operating conditions of the apparatus assumed, j_T is the greater, the greater Ga and the smaller Re, Pr, and E5 for the same length of the spiral.

In the stabilization region δ_T and β_T take the form

$$\delta_T = \sqrt[3]{\frac{45\delta^2 xa}{q\left(6 - \dfrac{\tau_0\delta^2}{\mu q}\right)}},$$
$$\beta_T = 0.82\,\frac{a^{2/3}q^{1/9}\omega^{4/9}A^{2/9}}{L^{1/3}\nu^{2/9}}\,f(\tau_0), \tag{16}$$

$$f(\tau_0) = \sqrt[3]{\frac{1 - \dfrac{\tau_0\delta^2}{6\mu q}}{\left[\exp\left(-\dfrac{0.26\tau_0}{\rho\omega^{4/3}A^{2/3}q^{1/3}\nu^{1/3}}\right)\right]^2}} \tag{17}$$

NOTATION

u, v are the projections of the velocity vector $\overline{V}$ on the x and y axes;

p is the pressure in the liquid film;

h_0 is the initial thickness of the liquid film;

$q = V_0 h_0$ is the rate of liquid flow;

U is the characteristic velocity;

$\overline{\delta} = \delta/h_0$, $\overline{\delta}_T = \delta_T/h_0$ are the dimensionless thicknesses of the hydrodynamic and thermal boundary layers;

$B = \tau_0/\mu = \partial u/\partial y$ is the frictional force at the surface of the liquid film;

$\overline{B} = Bh_0/V_0$ is the dimensionless frictional force at the surface of the liquid film ($\overline{B} = \Gamma 1\,\mathrm{Ga/Re}$, where $\Gamma 1 = 3\tau_0/\rho\omega^2 A h_0$);

ω is the angular velocity of rotation of the spiral;

A is the characteristic of the Archimedes spiral ($r = A\theta$);

$x = A[(\theta/2)\sqrt{\theta^2+1} + {}^1/_2 \ln(\theta + \sqrt{\theta^2+1})]$ is the current length of the spiral;

L is the characteristic length of the spiral;

$E5 = h_0/A$ is the dimensionless characteristic of the spiral;

$Re = 3V_0h_0/\nu$ is the modified Reynolds number;

$Ga = \omega^2 A h_0^3/\nu^2$ is the Galileo's number;

$E1 = \delta/h_0 = \sqrt[3]{Re/Ga}$ is the ratio of the thickness of the boundary layer to the initial thickness of the liquid film;

a is the thermal diffusivity;

$Pr = \nu/a$ is the thermal Prandtl number;

$\overline{x} = x/h_0$ is the dimensionless current length of the spiral.

LITERATURE CITED

1. N. S. Mochalova, L. P. Kholpanov, and V. Ya. Shkadov, Inzh.-Fiz. Zh., 25, No. 4 (1973).